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A Peculiar Phenomenon and its Potential Explanation in the ATP Tennis Tour Finals for Singles

Abstract

The ATP finals is the concluding tournament of the tennis season since its initiation over 50 years ago. It features the 8 best players of that year and is often considered to be the most prestigious event in the sport other than the 4 grand slams. Unlike any other professional tennis tournament, it includes a round-robin stage where all players in a group compete against each other, making it a unique testbed for examining performance under forgiving conditions, where losing does not immediately result in elimination. Analysis of the distribution of final group standings in the ATP Finals for singles from 1972-2021 reveals a surprising pattern, where one of the possible and seemingly likely outcomes almost never materializes. The present study uses a model-free, optimization approach to account for this distinctive phenomenon by calculating what match winning probabilities between players in a group can lead to the observed distribution. Results show that the only way to explain the empirical findings is through a "paradoxical" balance of power where the best player in a group shows a vulnerability against the weakest player. We discuss the possible mechanisms underlying this result and their implications for match prediction, bettors, and tournament organization.

Keywords: betting, model-free approach, round-robin, tennis

1. Introduction

The use of data analytics in professional tennis has become increasingly more common over the last decade. More and more players are taking advantage of advanced statistical analyses to improve their game, whereas media outlets, helped by software giants (e.g. I.B.M) use the data to present the audience with enriched analysis of matches in real time (Larson and Smith 2018). These analyses, however, have been mostly limited to the development of models and metrics to describe broad aspects of the game, such as predicting the outcomes of tennis matches (e.g., Ingram 2019; Klaassen and Magnus 2003; McHale and Morton 2011; Spanias and Knottenbelt 2013; see review by Kovalchik 2016), revising ranking systems (e.g., Bozóki, Csató, and Temesi 2016). ; Irons, Buckley, and Paulden 2014), or settling popular disputes with interest to pundits and general audiences (e.g., Radicchi 2012). While a few studies have examined more specific aspects, such as success rates of elite players in elite tournaments (e.g. Gallagher, Frisoli, and Luby 2021; Leitner, Zeileis, and Hornik 2009; Wei, Lucey, Morgan, and Sridharan 2013), their approach remained top-down: developing a model and then applying it to a particular dataset chosen for its prominent status and visibility.

Much less common are bottom-up approaches, which begin with identifying local, unique statistical patterns in the field, and then examine whether they could be accounted for by mechanisms that have broader implications on the sport. The current work attempts to illuminate such a unique pattern appearing in the ATP Finals tennis tournament for singles, explain its possible sources through a “model-free” statistical approach, and draw conclusions with possible interest to players, ATP officials, tennis pundits and betting agencies. To the best of our knowledge, this is also the first academic attempt devoted specifically to identifying and explaining statistical patterns in the ATP Finals in tennis.

The ATP finals (please see *https://www.nittoatpfinals.com/en/heritage/history*) is the concluding tournament of men's tennis calendarial year, organized by the Association of Professional Tennis (ATP). It has been taking place regularly since 1970, usually during November (and, at some years in its first decade, during January of the following year), and features the 8 players with the highest seeds in the ATP ranking based on performance during the season. Given that only the players with the best results of that year are allowed to participate, it is often considered to be the most prestigious tennis tournament other than the four Grand slams.

The ATP Finals are distinct from all other ATP-tour tennis tournaments in that they are not entirely designed as a knockout system where every match ends up in the elimination of the losing player. Instead, it employs a round-robin stage where the 8 players are organized into two 4-player groups, with each player playing against all other three in his group. Groupings are based on the players' seeding in an attempt to keep each group at a roughly similar level. As such, the number 1 and 2 seeds will find themselves in opposite groups, as will the number 3 and 4 seeds, 5 and 6, etc. The two players ending up on top of each group by the end of the round-robin stage then proceed to a regular knockout stage with semifinals (in which each group's winner faces the runner-up of the other group), followed by a final. Thus, during the group stage, a player may lose a match – or even two – and still remain in the tournament. The two winners of each group are determined by ordering their performance based on the number of wins, and, in case of a tie, based on a cascade of additional measures, including number of games played (mostly relevant in cases when a player skips an entire match due to injury and needs to be substituted), sets won, head-to-head results and so on (see *https://www.nittoatpfinals.com /en/event/rules-and-format*). Over the years, there have been several variations from this setup, particularly during the first decade and a half of the tournament's life. For example, in 1970-

1971 the tournament only had a group stage with one single group and no knockout stage whereas during 1982-1985 it was conducted as a regular knockout tournament from start to finish (*https://www.nittoatpfinals.com/en/heritage/results-1970-1999)*; the finals have sometimes been played as best-of-five sets rather than best-of-three like the rest of the tournament (*https://www.nittoatpfinals.com/en/heritage/results-2000-2021)* ; the allocation of players to groups based on seedings was not followed in the first few years; and the specific cascade of tiebreak rules determining the ranking of each group in case players end up with the same number of wins has seen some minor variations; but overall, the format has been quite stable for over half a century of the tournament's existence.

Our main concern in the current study is the peculiar statistical distribution of the final standings of the group stage in the ATP Finals in singles, particularly as they relate to the number of wins/losses. Since all 4 players play against each other for a total of 6 matches and there is always one winner and one loser in a tennis match, the final standings can result in only one of four outcomes[1]:

(i) One player winning all of his matches, one player winning two matches, one player winning one match and one player winning none (3-2-1-0).

(ii) Two players winning two matches and two players winning one match (2-2-1-1).

(iii) Three players winning two matches and one player winning none (2-2-2-0).

(iv) One player winning three matches whereas the other 3 winning one match each (3-1-1-1).

The frequency of each of these outcomes could enlighten us about the balance of power between players in an ATP Finals group. For example, if we assume the (highly unlikely) scenario where all players are equally strong with each having a probability of exactly 0.5 to win

[1] Here, we disregard the (somewhat uncommon) situation where a player gets injured and is substituted by another player mid tournament, and treat it as if the same player was playing throughout. This is discussed later on.

a match, it can be shown that the expected frequency distribution of the final standings over the 4 possible outcomes will be [0.375 0.375 0.125 0.125]. In other words, it would be as likely to find a 3-2-1-0 and a 2-2-1-1 standings, with each occurring 3/8 of the time, and it would be as likely to find a 2-2-2-0 and a 3-1-1-1 standings, with each occurring 1/8 of the time.

Naively, one would assume that each of the possible outcomes would be at least somewhat likely; however, when examining the standings over all groups across the years of the ATP Finals' existence, we find a highly skewed distribution, with the fourth outcome (3-1-1-1) occurring in only 2 out of 92 cases, a frequency of merely 0.0217 (for comparison, in the ATP Finals for doubles, played in a similar format, no outcome occurs less than 0.085 of the time; and in the WTA Finals, the equivalent tournament in women's tennis, all outcomes have frequencies above 0.105);. Having such a low probability for a seemingly reasonable scenario (especially given that the other scenario where one player wins 3 matches, 3-2-1-0, is very likely) deserves explanation, as it may defy expectations set not only before the beginning of proceedings but also while the tournament is already underway (with potential repercussions to betting patterns). The following study attempts to explain this finding by examining what balance of power among the players in a group, as expressed by their probabilities of winning a match against each other, could result in such a peculiar group standing distribution.

2. Methods

Data on all match results in the ATP finals for singles were extracted from the official ATP website (https://www.atptour.com/en/scores/results-archive). Relevant data included 46 out of the 53 years of the tournament's existence (1972-1981 and 1986-2021), when it was played with a round robin stage that included two 4-player groups. In addition, when we needed to determine the exact order of matches played, information was extracted from the sports statistics website Flashscore (https://www.flashscore.com/).

We begin the analysis by computing the frequency of each of the 4 possible outcomes of the group standings. Each tournament contributes two samples for the calculation (corresponding to the two groups in each year), resulting in 92 samples over 46 years. The resulting distribution was:

$$\vec{T} = [0.6739\ 0.2065\ 0.0978\ 0.0217] \qquad (1)$$

for the 3-2-1-0, 2-2-1-1, 2-2-2-0 and 3-1-1-1 outcomes, respectively. $\vec{T}$ is thus considered as the empirical target distribution that we aim to explain in this study.

Next, we characterize the results of a round-robin group by $\vec{p}$, a vector with 6 values ($p_1 \ldots p_6$) representing the probabilities of a win by one player over another, which can be displayed in the following matrix form:

Winning player \ Losing player	1	2	3	4
1		p_1	p_2	p_3
2	$1-p_1$		p_4	p_5
3	$1-p_2$	$1-p_4$		p_6
4	$1-p_3$	$1-p_5$	$1-p_6$	

We search for a value of $\vec{p}$ that yields the target distribution of the group standings outcomes. For simplification, we assume that the results of each of the 6 matches taking place in an ATP Finals group in any year are independent of each other (i.e., each match depends on the relative contemporary strength of the players involved, but not on the results of the other matches in the group or any other results). While this is not necessarily the case, it is a reasonable approximation for which some support is given later on. Again for simplicity, we ignore instances where substitute players were used due to one or more players getting injured and forced to quit the tournament, and treat them like any other sample. Substitutions are an uncommon though not negligible phenomenon, occurring in 12 out of the 92 cases; however, they do not change the basic win-loss statistics we target in this study and our conclusions are valid even when discarding them from the calculations, therefore we report the results with all data included. Finally, note that our approach intentionally ignores seedings since our goal is to describe the patterns over all groups in the tournament's history with no prior assumptions about likely results based on previous performance in a given year. Seedings are addressed only at one point when trying to estimate the stability of the statistics (see Results).

To estimate $\vec{p}$, we search for values of $p_1 \ldots p_6$ that yield a distribution of group standings $\theta(\vec{p})$ that is as close as possible to the target distribution. We define the distance between the two distributions based on the Kullback-Leibler divergence (D_{KL}; Kullback and Leibler 1951), which gets a value of 0 when the two distributions are identical, or a positive value when they are not.[2] This turned the calculation into an optimization problem, where our goal is to find $\vec{p}$ that minimizes the objective function D_{KL}:

$$\underset{\vec{p}}{\operatorname{argmin}}\, D_{KL}\left(\vec{T} \parallel \theta(\vec{p})\right) \quad (2)$$

[2] Similar results are achieved when using other objective functions to define the difference between the two distributions, such as sum squared difference, City block distance (the sum of absolute differences in each dimension), or the counterpart definition for Kullback-Leibler divergence, $D_{KL}\left(\theta(\vec{p}) \parallel \vec{T}\right)$. D_{KL} was preferred because it more naturally captures similarities between distributions and thus requires fewer repetitions to effectively cover the parameter space.

A small additional correction is applied to the objective function due to the inherent limitations on precision when using a finite amount of available data. Specifically, substantially different values of $\vec{p}$ can bring the objective function close to 0 with only tiny disparities that do not meaningfully reflect a higher likelihood of one set of $\vec{p}$ values over the other. To overcome this, we set a threshold for the difference between $\theta(\vec{p})$ and $\vec{T}$, below which the objective function was manually set to 0. The difference was computed as the City block distance (see footnote 2) between the two distributions, and the threshold was determined to be 1/92, the resolution of the target distribution (given 92 data points, a City block distance between $\theta(\vec{p})$ and the target distribution that is higher than 1/92 suggests that $\theta(\vec{p})$ is, in fact, closer to another target distribution that could have been produced with the same amount of data).

Estimation of $\vec{p}$ was performed numerically using the Nelder-Mead simplex algorithm. The optimization algorithm was run on Matlab 2021a (Mathworks) using the built-in *fminsearch* command. Since the algorithm's output is sensitive to initial conditions, we repeated the optimization procedure 50,000 time, each time starting with a random initial condition $\vec{p}_0$, to assure a good coverage of the whole parameter space (additional runs did not change the results much further, nor did dividing the parameter space into an evenly spaced grid and setting the initial conditions to each of the grid edges). Other analyses described in Results, including Principal Component Analysis (PCA; Abdi and Williams 2010) and K-means clustering (Lloyd 1982) were carried out using the Matlab commands *pca*, *kmeans* and *kmeans_opt*.

3. Results

We first tested our assumption that the group standings in tournaments are approximately independent of each other. To that end, we computed the joint probability distribution of the two group standings of each year (across the 46 years of available data), which includes 10 possible outcomes (all pair combinations of the 4 possible final standings, disregarding order; for example, one outcome is when the two groups in a single year both end with 3-2-1-0 standings; another is when one group ends with 3-2-1-0 and the other with 2-2-1-1; and so on). We then computed the expected frequency of the joint distribution had the two groups been equally and independently distributed, using the target probability *T* extracted from the full dataset. These two joint distributions are presented in Figure 1, ordered by the magnitude of the expected frequency of each outcome.

As can be seen, with minor exceptions, the two distributions resemble each other considerably. This was confirmed using a chi-square goodness of fit test comparing the two distributions (multiplied by 46, the number of data points), yielding a non-significant value ($\chi^2(3, N = 46) = 0.881$, $p = 0.83$)[3]. While this test is not a strong guarantee that the groups are indeed independent (given the limited data), it serves as a sanity check to confirm that this approximation is not completely unrealistic.

---- Place Figure 1 Here ----

We further evaluated the assumption of equality and independence of the final group standings each year by distinguishing the groups based on players' seeding. While there is no a-priori "correct" way to differentiate between the two groups of each year's tournament as if

[3] Given that many of the possible outcomes yield an expected count that is smaller than 5, the minimum value required for reliably applying a chi-square test, we pooled together the 7 least-frequent outcomes into one big category, yielding a total of 4 outcome categories used in the statistical test. We also verified this result by running Fisher's Exact test on the full 10 categories (since this test requires integers representing exact number of occurrences, the values of the expected frequency were rounded). The result showed, again, that the two distributions were not significantly different ($p = 0.97$).

representing samples of two different variables, differentiating by seeding presents a natural and appealing option since in the majority of years, there has been a deliberate attempt to maintain a roughly equal draw by making the groups as equal as possible in respect to their seedings (as mentioned earlier in Introduction). We therefore differentiated between the groups that included the number 1 seed ("Group 1") and the groups that included the number 2 seed ("Group 2") and separately analyzed their performance over the years. Only 39 of the 46 years of available data were included in this analysis since for 7 years groups were not equaled based on seedings. We examined three measures of performance for each group: (1) group standings distribution over the years; (2) probability of the players in the group reaching the finals; and (3) probability of the group yielding the eventual winner of the tournament.

We found that the distribution of group standings for Group 1 was [0.6667 0.1795 0.1026 0.0512] whereas for Group 2 it was [0.6410 0.2308 0.1282 0]. Fisher's exact test showed there was no significant difference between the two ($p = 0.62$), nor was there a difference between each of them and the target distribution $\vec{T}$calculated over the entire data (both p's> 0.85). The probability of a player from one of the groups reaching the final was 0.526 for Group 1 and 0.474 for Group 2, and the probability of a player from one group winning the tournament was 0.538 and 0.462 for Group 1 and 2, respectively. Fisher's exact test showed, again, that neither difference was significant (both p's>0.78). In summary, when differentiating the groups based on seedings and separately evaluating each group's performance over the years, we found that they exhibit roughly the same performance overall, with similar distributions of final group standings and success in yielding the finalists and winner of the tournament.

Having verified that the preliminary assumptions of our approach are acceptable, we next moved to perform the main analysis of fitting a value for $\vec{p}$, the vector of win probabilities of each match in a group, using the numerical optimization procedure described in Methods. The calculation produced a variety of solutions for $\vec{p}$ reflecting a range of values that perfectly minimized the objective function (up to the possible precision point; see Methods). The range for each $p_1 \ldots p_6$ values across the 10,297 perfect solutions found is displayed in a color chart in Figure 2, corresponding to the winning-losing player matrix presented in Methods. Rows were ordered from the best player in the group (top) to the weakest player (bottom), and the range of values for each p_i is displayed within the corresponding cell sorted from the highest (center) to lowest (edges).

---- Place Figure 2 Here ----

As is evident in Figure 2, across the range of possible values, the strongest player in the group was always highly likely to win against the 2nd- and 3rd-best players (with probabilities that are predominantly between 0.85-1), while the 2nd-best players almost always won against the weakest player in the group (with a probability that is close to 1). The matches between the 2nd- and 3rd-best players, as well as the match between the 3rd-best and the weakest player, were more varied with probabilities that predominantly ranged between 0.6 and 1. With one notable exception, the probabilities for a win generally tended to have the expected pattern of becoming higher and higher for each player as they faced the weaker players of the group, evident by an overall increase in values in each row from left to right. The one notable exception was the match between the strongest and weakest players in the group (upper right and bottom left cells): For every possible solution, this match never favored the best player decisively, with probabilities that barely reached 0.8 and were most often closer to 0.75 or lower. In other words, the optimization analysis led to a range of results with one peculiar core theme: *a relative* *weakness of the best player in the group when facing the weakest player.* One additional peculiar result was the absolutely dominance of the 2nd-best player over the weakest player. While an advantage is expected, this was the most one-sided matchup in the whole matrix (higher, for example, than any of the winning probabilities of the best player), and it remained uniformly high for all possible solutions.

---- Place Figure 3 Here ----

To gain deeper understanding of the core structure of the results, we performed PCA over the various solutions for $\vec{p}$. We found that the majority of the variance in the solutions lied in the first principal component, indicated by the first eigenvalue being more than 5 times larger than the 2nd eigenvalue, and more than an order of a magnitude larger than the rest of the eigenvalues (Figure 3A). This first principal component almost exclusively modulated the probabilities of the 3rd-best player's matches. As seen in Figure 3B, the most affected probabilities were p_4 and p_6, representing the probability of the 3rd-best player losing to the 2nd-best player and winning against the weakest player, respectively. This influence was almost equally strong and in the opposite direction: The more likely the 3rd-best player was to lose to the 2nd-best player, the less likely he was to win against the weakest player. To a lesser degree, the probability of the 3rd-best player to lose to the best player (p_2) was also influenced, in the same direction as his probability to lose to the 2nd-best player. So, in essence, the main variability in the solutions expressed the level of play exhibited by the 3rd-best player: from being closer in level to the 2nd-best player (and as a consequence a bit closer to the best player) on one end to being closer to the weakest player on the other end. Other than that, the balance of power between the players was quite stable (given the low values of the remaining eigenvalues) and could be expressed by the average values of $\vec{p}$ across the different solutions. In a matrix form, $\bar{P}$ was equal to:

Winning player \ Losing player	1	2	3	4
1		0.952	*0.882* *(-0.5x)*	*0.757*
2	0.048		*0.865* *(-x)*	*0.988*
3	0.118 *(+0.5x)*	0.135 *(+x)*		*0.857* *(+x)*
4	0.243	0.012	0.143 *(-x)*	

Here, the average values are portrayed in the center of each cell. To allow easier comprehension of the possible variability in the level of the 3rd-best player, we also express, in brackets, the range of values resulting from adding the contribution of the first eigenvector. This is done using a variable, x, which could assume any value in the range [$-0.135 < x < 0.143$]. So, for example, the probability of the 3rd-best player winning against the weakest player could range from 0.722 (when $x = -0.135$) to 1 (when $x = 0.143$), with x simultaneously affecting the remaining 3rd-best player's winning probability against the other players.

Disregarding all eigenvectors, the average $\bar{\vec{p}}$ alone was enough to yield a group standing distribution $\vec{T}'$ that was a pretty close fit to the target distribution ($\vec{T}'$= [0.6622 0.2178 0.0912 0.0288]; compare to equation (2)), proving that the variety in solutions, while mainly reflecting different possible strengths of the 3rd-best player compared to his opponents, did not contribute much in determining the group standings distribution. The $\bar{P}$ matrix above therefore represents the core balance of power between players in the ATP Finals for singles that lead to the empirical target distribution, which is the solution we were aiming to achieve.

Next, to examine how the match winning probabilities affected the eventual group standing distribution, we fluctuated each of the 6 probability values while keeping the others constant at their average value and calculated the resulting distribution. Results are displayed in Figure 3C. As can be seen, the low frequency characterizing the 3-1-1-1 outcome is most strongly determined by the superiority of the 2nd-best player over the weakest player, as represented by p_5; diminishing this superiority quickly increases the frequency of that outcome. In contrast, the vulnerability of the strongest player when facing the weakest player (represented by p_3), is a major influence on the 3-2-1-1 and 2-2-1-1 outcomes. If the strongest player did not have this vulnerability, the outcome distribution would have been even more skewed than it is, with almost all groups ending with a 3-2-1-0 outcome.

To conclude the analysis, we examined how "natural" match winning probabilities would influence the group standings. We define natural probabilities as those that unambiguously reflect systematic differences in the level of play between players in a group. Specifically, the best player would have a higher than 0.5 chance to win against any other player in the group with his winning probability values assuming an ascending gradient: The lowest probability would be against the 2nd-best player and the highest probability would be against the weakest player. Likewise, the 2nd-best player would have a higher than 0.5 chance to win against the 3rd-best and weakest player, with the latter probability being higher than the former, and both probabilities being lower than the corresponding ones for the best player when playing against the same opponents; and so on (in other words, the "natural" probably matrix, in contrast to the $\bar{P}$ matrix, will have increasing values from left to right in every row, and decreasing values from top to bottom in every column).

To investigate the outcome of such settings, we randomized 10,000 $\vec{p}$ values under the above constraints and calculated the resulting group standings. Figure 4 displays 15 prototypes of these group standing distributions, obtained by running an optimized K-means clustering analysis on the 10,000 samples (see Figure caption for details). As expected, none of the prototypes resembled the target distribution (Figure 4, top left panel), and particularly none reflected the extremeness of the 3-1-1-1 outcome frequency. When looking at individual distributions, we found that only 12 out of the 10,000 (0.12%) resulted in the same or lower frequency of the empirical 3-1-1-1 outcome, showcasing how unsuitable the natural probabilities are for producing the target distribution. Moreover, in all cases where the 3-1-1-1 frequency was low, the 2-2-1-1 frequency was low as well (always below 4%) while the winning probabilities of the top 3 players against the weakest player was very high (all above 91% with the majority of cases being at 97% or higher). In other words, the natural probabilities produce a low 3-1-1-1 frequency only when the weakest player was barely able to win a single match – necessarily making the 2-2-1-1 frequency low as well. This is partly similar to the distribution depicted in Figure 3C, third panel from the left, when p_3 is assuming high values. To summarize, the target distribution, characterized by both a very low frequency of the 3-1-1-1 outcome and a medium frequency of the 2-2-1-1 outcome, cannot be achieved by winning probabilities that reflect a simple gradient in the level of play in a group. To explain the target distribution, a "non-natural" element needs to be introduced, such as the vulnerability of the strongest player in the group to the weakest player.

---- Place Figure 4 Here ----

4. Discussion

4.1 Summary and interpretation of the main results

Our goal in this study was to uncover which balance of power between players in the singles tournament of the ATP Finals can lead to the observed skewed distribution of the tournament's round-robin group standings, where one outcome is, surprisingly, extremely rare. Using a simple "model-free" approach that assumes stationary statistics of the match win probabilities between the players, we found a specific stable pattern that characterizes this balance of power, as displayed in the $\bar{P}$ matrix. We can sum up the core elements of this pattern as follows:

1. One player is an overwhelming favorite to win the group, showing clear dominance over all other players; and another player is an obvious underdog with low probability to win against the others.
2. Despite his superiority, the favorite player has nevertheless a relative vulnerability when facing the underdog (exemplified by his lower probability of winning that match compared to his other two matches against superior players)
3. The 2nd-best player totally dominates the underdog and has an advantage over the 3rd-best player, which can be big or small (reflecting the 3rd-best player's general level)

Although our results uncover the balance of power that can yield the empirical group standing distribution, the reason why this balance of power appears in the first place demands explanation. Specifically, it is worth discussing what could yield our most notable finding, the fact that the underdog has a relatively high chance to surprise the favorite while still being totally dominated by the 2nd-best player. It may be tempting to view this result as a general example of a "puncher's chance" (the phenomenon by which an underdog occasionally defies the expected odds and beats a much stronger player; e.g., Holmes, McHale, & Żychaluk, 2022); however, a more direct explanation could arise from one specific procedure followed by the ATP Finals concerning the way the order of matches is determined. The round-robin of the ATP Finals is organized such that the winners (and the losers) of the first two matches always meet in their second match. For example, if the first 2 matches in a group were played between players A and B, with A winning, and between C and D, with D winning, the next two matches would be between A and D, and C and B. That order, on its own, increases the probability that the match between the strongest and weakest players in a group would be the last one. Assuming that the pairing of players in their first match is totally random, it can be shown, using the $\bar{P}$ matrix (with x=0), that the favorite and underdog players would meet in their last match in 56% of the times (as compared to a baseline of 33% if the order of all matches was totally random). In reality, the rules regarding the initial pairing have slightly fluctuated over the years based on players' seeding in a way that could either increase or decrease this probability; but an empirical examination shows that among all groups that ended in either a 3-2-1-0 or 2-2-1-1 outcome (the outcomes that, as described above, depend the most on the result of the match between the favorite and the underdog), the match between the winner of the group and the loser of the group was the final one in approximately 59% of the cases[4]. Assuming the winner and loser of the groups that end with these outcomes are the strongest and weakest players, respectively (an assumption that is obviously not always true, but quite often is; see the $\bar{P}$ matrix), this result

[4] This result was calculated based on all ATP Final tournaments from 1990 and on, for which match order is readily available.

further supports the fact that the favorite and underdog in a group are more often than not meeting only in their final round-robin match.

The relevance of this result to our finding is quite straightforward: It suggests that the favorite often arrives to his last match having already won the previous two and after already qualifying to the semifinals. Such situations are known in sports to potentially lead to an intentional lack of effort, either to “save the body” for the matches ahead or simply due to a lack of interest in a match that doesn’t determine much. Consequently, the probability for the underdog to surprise the favorite increases, despite the significant gap in their base level. Importantly, this scenario does not apply – and, in fact, is opposite - to the experience of the 2^{nd}-best player. The 2^{nd}-best player would predominantly meet the underdog in either his first or second round-robin match, when each result is still crucial in determining the final outcome of the group. Therefore, the 2^{nd}-best player is expected to “give his best” in these matches, potentially resulting in the full expression of his advantage over the underdog.

4.4 Limitations of the approach

One assumption taken in this study is that the match-winning probabilities in the ATP Finals are stationary. There exists, however, a finding that casts some doubt on this assumption. It concerns another unique characteristic of the ATP Finals: The potential of two players meeting more than once. This situation can occur in only one scenario: When the top two players of one group end up meeting again in the final (after having won their respective semi-finals). Ostensibly, we would expect the outcome of both matches to be the same more often than not, reflecting the relative strength of the two players and assuming stationary statistics. However, in the 19 times this scenario has played out in the ATP finals, more than half (11 times, or ~58%) resulted in a switch of the winner's identity. This result cannot be accounted for by any stationary balance of power between the players in the round-robin stage. It is also quite peculiar on its own merit, given that the two matches are played under similar conditions, only a few days apart. It implies that the two matches likely have a different winning probability – in other words, they reflect non-stationary statistics. This peculiar finding could be partially accounted for by observing that in the majority of years where such repeated encounter has taken place, the final – and only the final – was played in a best-out-of-5 sets format, rather than best-out-of-3 like the remainder of the tournament. Best-out-of-5 matches tend to emphasize some aspects of game play that are less important in best-out-of-3 matches, such as stamina and endurance. While not fully explaining the switch in the winner's identity (after all, we would still expect the better player to show his superiority despite the difference in match length), this observation at least serves to highlight why the winning probability may not be stationary. However, even when excluding the years when the final was played in a best-out-of-5 format, we still find that the player who won the round-robin encounter proceeded to win against the same opponent in the final in only 4 out of 7 times (57.1%, which is far less than the expected probability of 75.7% portrayed in the $\bar{P}$ matrix). It is difficult to draw strong conclusions from such limited amount of data, but it does seem that, overall, repeated matches in an ATP Finals tournament lead to a modification in the winning probability between the players involved, whether played in a best-out-of-5 or best-out-of-3 sets. Potential accounts for this result (e.g., adjustments of the losing player following the first game that improve his chances considerably given the opportunity to face the same player again within a short period of time under the same conditions) will need to be explored in future studies.

4.5 Implications and Conclusions

Several conclusions can be drawn from our study.

First, our results exemplify the type of non-trivial, tournament-specific information that should be taken into account when considering betting odds in tennis (or other sports for that matter). For example, consider the final pair of matches played in a round-robin stage in the ATP Finals. Knowing that a 3-1-1-1 outcome is so rare, one could exploit this information when placing bets on these matches even if it contradicts more immediate information about the identity of the players involved and their seedings. Indeed, over the last 3 decades, there have been 8 occasions where a 3-1-1-1 outcome was one match away from materializing with the result of this match only needing to follow the players' seeding (often the more likely result in betting agencies). However, in only 2 of those times did the "likely" result occur. In other words, the 3-1-1-1 outcome has such low a-priory probability that, even when it depends on one final match going according to seeding, its conditional probability does not rise above 25%.

Second, our results highlight the degree to which decisions on the format and settings of a tournament affect outcomes. For example, the order by which matches are played will often have unpredictable effect on who is advancing to the next round and who is not; and whether players compete against each other only once or multiple times will have a strong influence on their probability to come out on top. Tournament directors and other stakeholders should be aware of such non-trivial dependencies when determining the rules and regulations of play, in tennis and otherwise.

Finally, our results serve to demonstrate how surprising, extreme or unexpected statistical phenomena in sports can serve as a fruitful platform to uncover underlying mechanisms in play, sometimes even negating the need for a complex statistical model. In our case, the peculiar distribution of the final group standings in the ATP Finals for singles, as well as the unique format of the tournament itself, contributed to our finding of a specific balance of power among players, one that may not be evident when looking at more standard or widespread settings. This approach, of looking at edge cases, resembles a common practice in fields like neuroscience, where aberrant states – for example, a patient with brain lesions that cause unique deficiencies in the perception of reality – can teach us a lot about the primary brain processes involved. Since most academic papers on tennis choose to address global patterns, across whole careers and multiple tournaments, they may miss statistical trends that could be more relevant for predictions of local events. Future studies may adapt our general approach to analyze other tournaments employing a round-robin stage, such as the WTA Finals, the Davis Cup and the Billie Jean King ("Fed") Cup, to potentially uncover their own unique statistical trends.

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Figure Captions

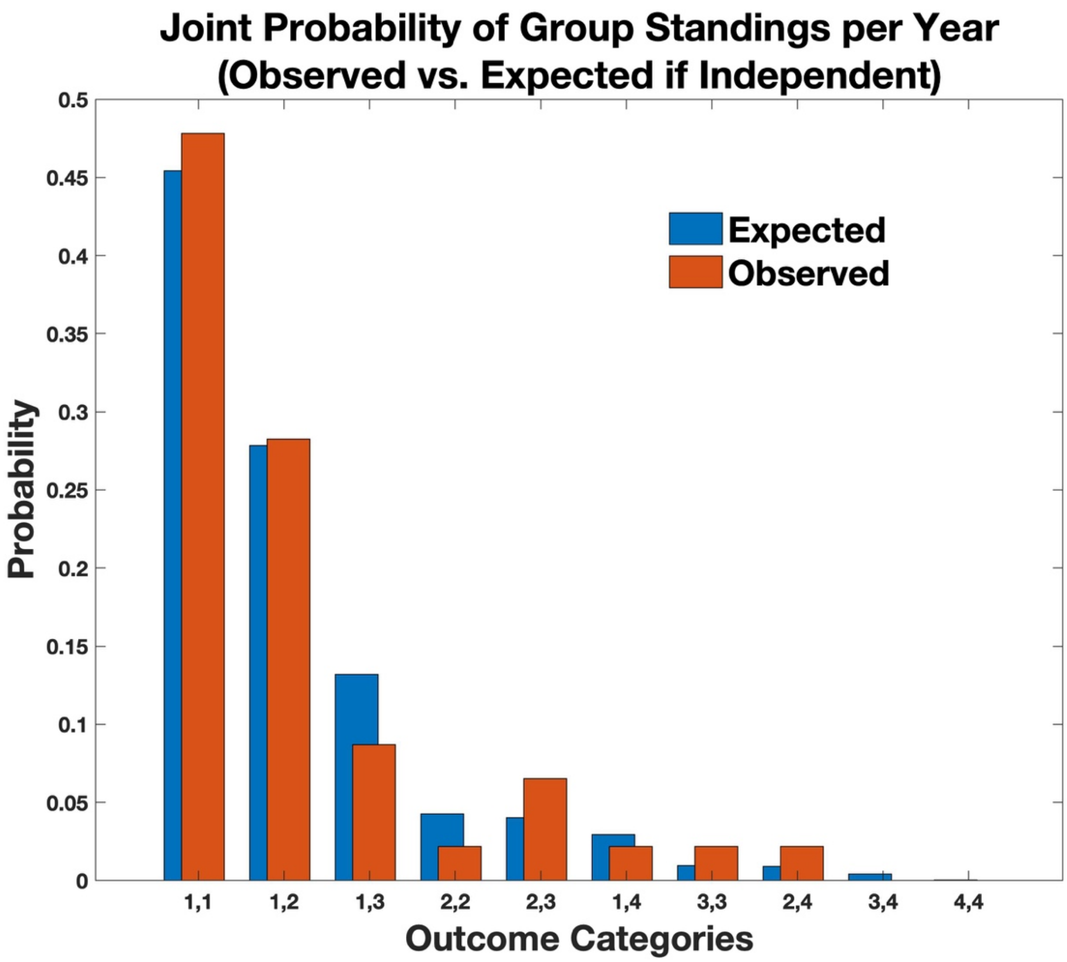
Figure 1: Comparison between the observed joint probability distribution of possible group standing outcomes for the two groups in the ATP Finals singles each year and the expected distribution if the groups were independent and equally distributed. Data is based on 92 groups over 46 years of the tournament. The numbers 1,2,3,4 on the x-axis refer to the four possible group outcomes, [3 2 1 0], [2 2 1 1], [2 2 2 0], [3 1 1 1], respectively.

Figure 2: Range of solutions for the match winning probabilities among players in the ATP finals round-robin.

Figure 3: Analysis of the match winning probability solutions. A: Eigenvalues corresponding to the 6 eigenvectors representing the variance of solutions for the 6 probability values following Principal Component Analysis (PCA), showing the 1st eigenvector is by far the most critical to describe the variety in solutions. B: Weights of the 1st eigenvector. Values for p_4 and p_6 (representing probabilities for the matches between the 3rd-best player against the 2nd-best and

weakest players, respectively) are the ones most influenced, in opposite directions. p_2 (representing the match between the 3rd-best and the strongest player) is also influenced, to a lesser degree. C: Modulation of the group standing distribution as a result of variations in each probability value from its average (see the $\bar{P}$ matrix; the averages are marked by black dots). Results show the 3-1-1-1 outcome is most strongly influenced by p_5, whereas the 3-2-1-1 and 2-2-1-1 outcomes are most strongly influenced by p_3.

Figure 4: Prototypes of group standing distributions resulting from "natural" winning probabilities (15 panels in blue bars; see text for the definition of natural probabilities in this context). The prototypes were identified using K-means clustering (Lloyd 1982), with the optimal number of clusters determined using the 'elbow' method (Kodinariya and Makwana 2013). The vertical black lines represent one standard deviation above and below the mean (covering about 65% of the individual samples contributing to the prototype). For comparison, the empirical target distribution is displayed in red bars on the top left panel.



Joint Probability of Group Standings per Year
(Observed vs. Expected if Independent)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Probability
Expected
Observed
1,1
1,2
1,3
2,2
2,3
1,4
3,3
2,4
3,4
4,4
Outcome Categories

Figure 1

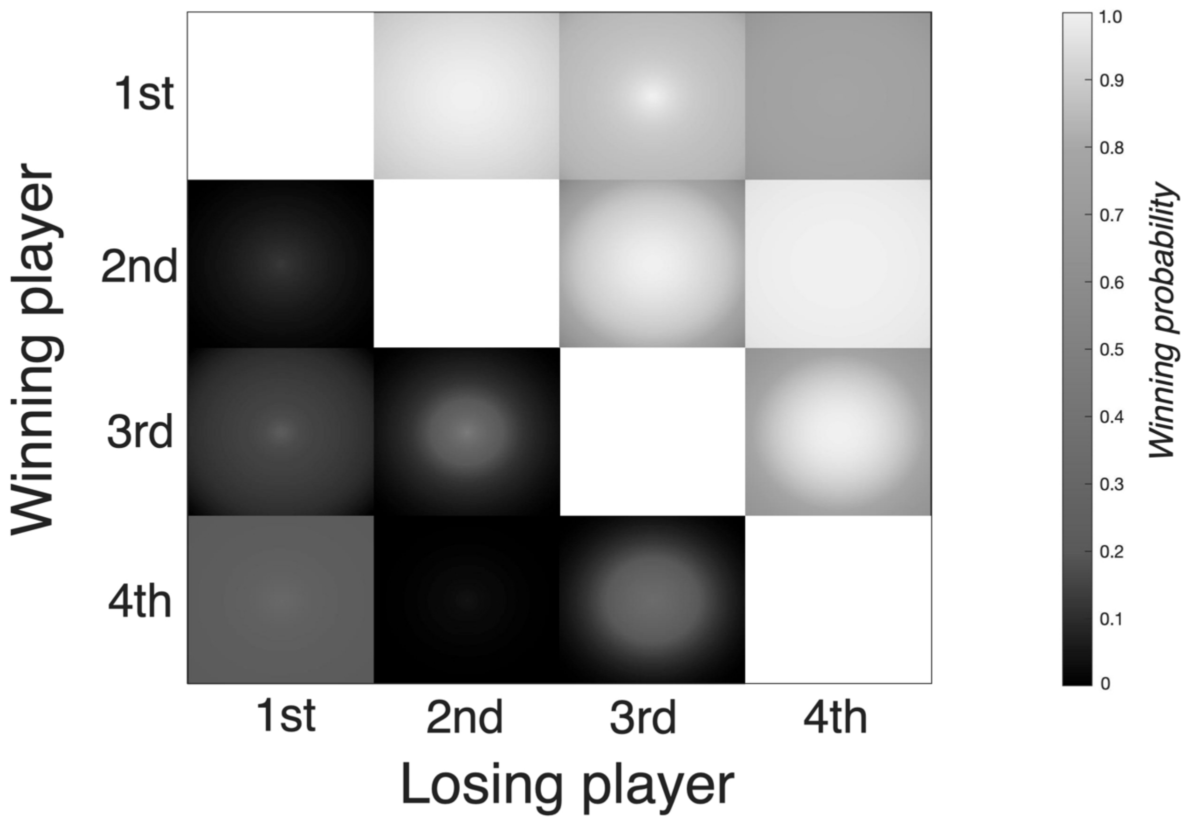

Winning player
1st
2nd
3rd
4th
Losing player
1st
2nd
3rd
4th
Winning probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Figure 2

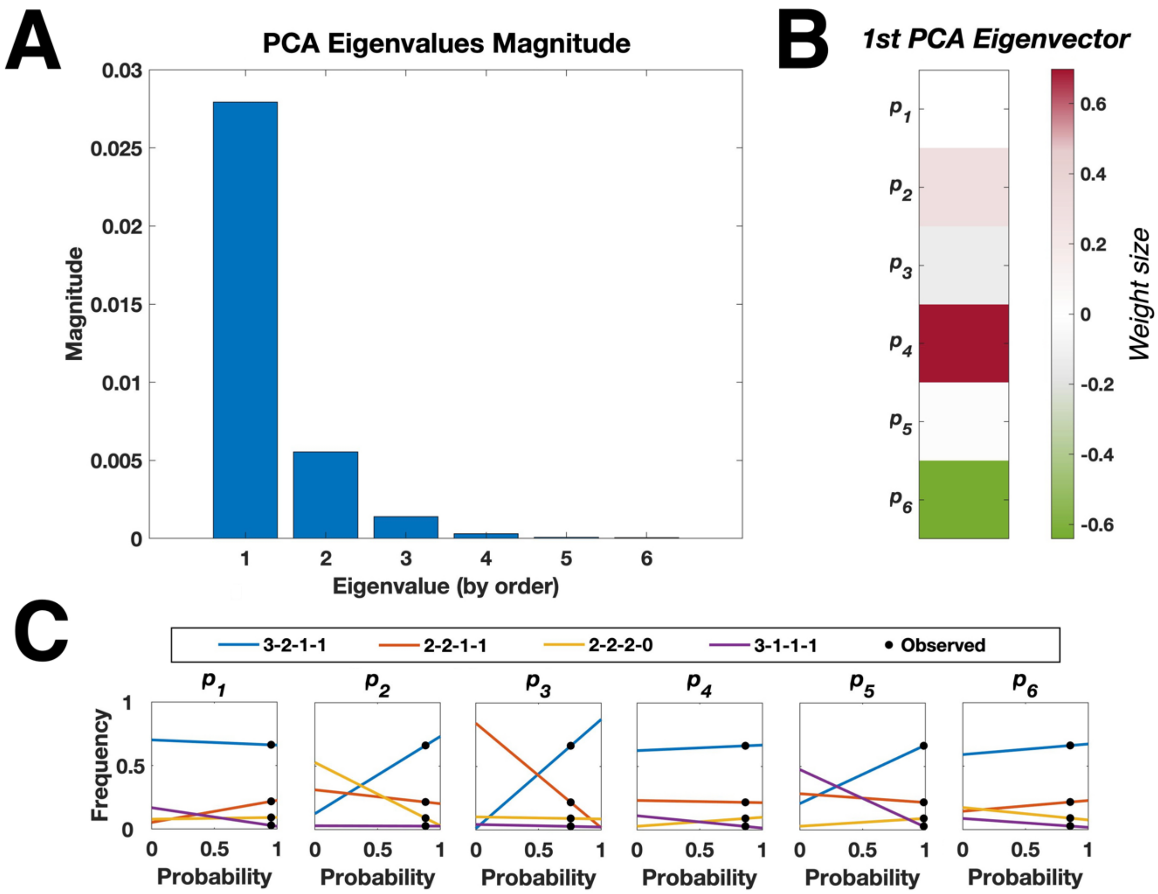

A
PCA Eigenvalues Magnitude
Magnitude
Eigenvalue (by order)
B
1st PCA Eigenvector
p_1
p_2
p_3
p_4
p_5
p_6
Weight size
C
3-2-1-1
2-2-1-1
2-2-2-0
3-1-1-1
Observed
Frequency
Probability

Figure 3

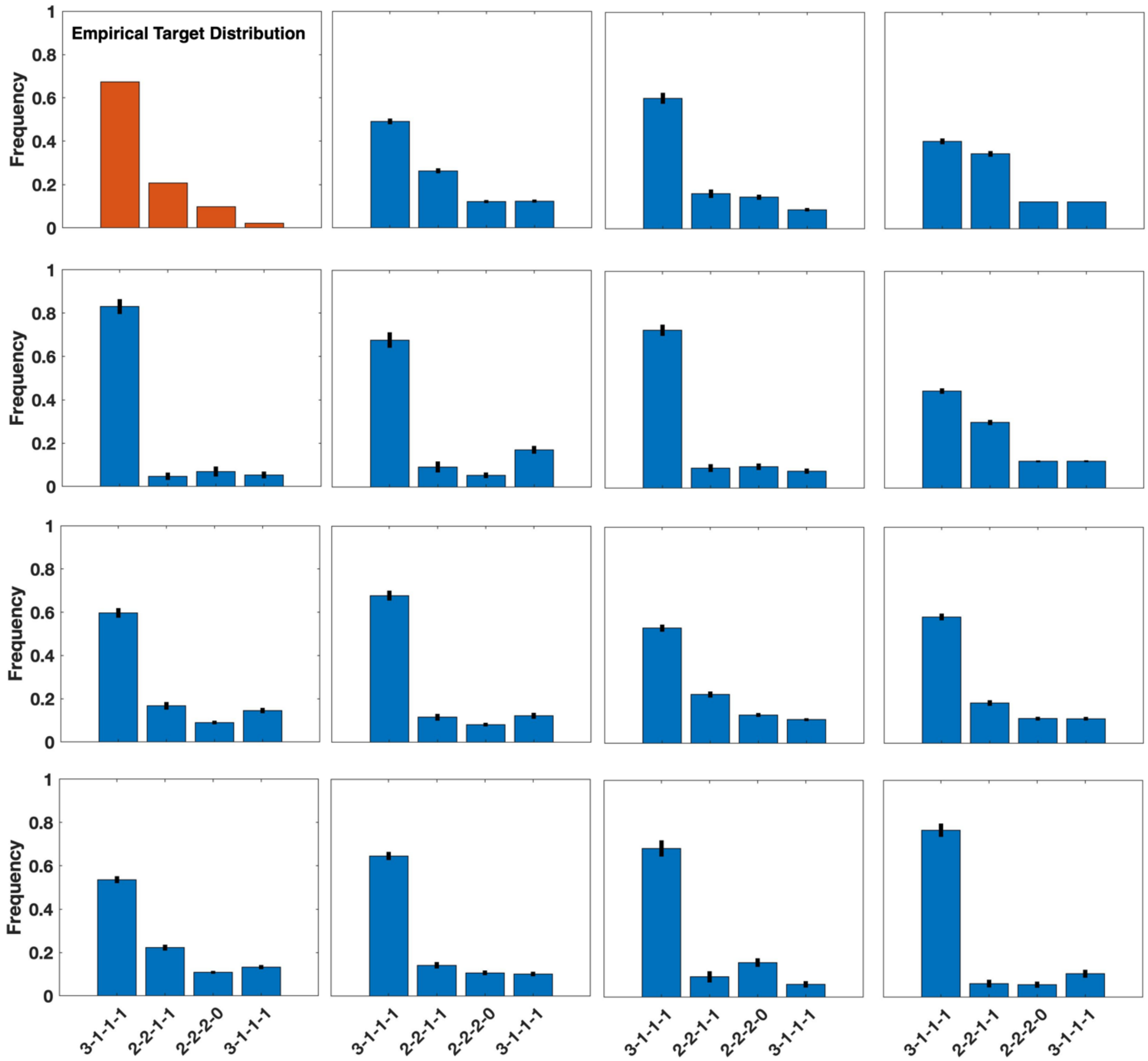

Empirical Target Distribution
Frequency
1
0.8
0.6
0.4
0.2
0
3-1-1-1
2-2-1-1
2-2-2-0
3-1-1-1

Figure 4